

1) HW #5 due Thursday

We were solving y'' + (t - i)y' + y = 0We assumed y(t)= Sant

and got $(t-1)y' = -\alpha_1 + \sum_{n=1}^{\infty} (n\alpha_n - (n+1)\alpha_{n+1})t$ り ー |

$$y'' = \sum_{n=0}^{\infty} (n+\lambda)(n+1)q_{n+\lambda} t^n$$

$$O = \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} t^{n}$$

$$+ \sum_{n=0}^{\infty} a_n t^{n} - a_1 (t-1)y'$$

$$+ \sum_{n=0}^{\infty} (na_n - (n+1)a_{n+1})t^{n}$$

$$Tsolate t^{n} term, combine all others: bad!
$$O = (2a_2 + a_0 - a_1)t$$

$$\sum_{n=1}^{\infty} (n+1)((n+2)a_{n+2} - a_{n+1} + a_n)t^{n}$$$$

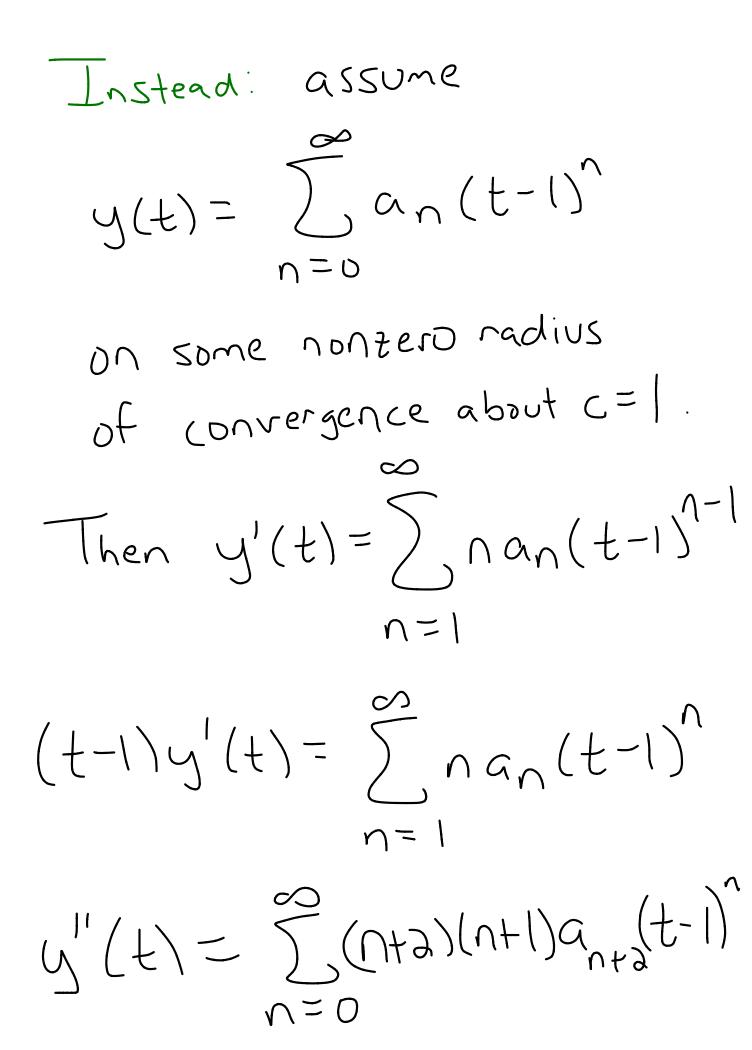
Seems difficult to create a pattern from

 $2a_{2} - a_{1} + a_{0} = 0$

 $(n+1)((n+2)\alpha_{n+2}-\alpha_{n+1}+\alpha_n)=0$

because there are three

Coefficients in each one.



M' $D = \sum_{n+2}^{n+2} (n+2) (n+1) a_{n+2} (t-1)^{n+2}$ **ハ**= 0 $+ \tilde{\Sigma} nan(t-1) + \tilde{\Sigma}, an(t-1)$ $(t-1)y^{1}$ Isolate (t-1) coefficient, combine other terms. $D = 2a_2 + a_{t} \sum (n+1)(a_{n+2}(n+3)+a_n)(t-1)^n$ トニト

 $D = 2a_2 + a_{t} \sum_{n+1}^{n+1} (a_{n+2}(n+3) + g_n)(t-1)^n$ nニ If equal to zero, coefficients

must equal zero, so

	$\Im_{a} + \alpha_{o} = 0, a_{d} = -\frac{\alpha_{o}}{\alpha}$
リニチ	$3(4a_{4}+q_{2})=0$ $a_{4}=-\frac{a_{2}}{4}=\frac{a_{0}}{2\cdot 4}$
n=4	5(696+94)=0 -64=-90 -64=-90 -64=-90 -64=-90
	$a_6 = \frac{-a_4}{6} = \frac{-a_4}{6} \cdot \frac{4}{6} \cdot \frac{6}{6}$

Assuming as is chosen,

$$a_{2K} = \frac{(-1)^{k} a_{b}}{2^{k} k!}$$

$$n=1:2(3q_{3}+q_{1})=0$$

$$G_{3}=\frac{-q_{1}}{3}$$

$$n=3 \quad M(5q_{5}+q_{3})=0$$

$$G_{5}=-\frac{q_{3}}{5}=\frac{q_{1}}{3\cdot5}$$

 $n=5: G(7a_{1}+a_{5})=0$ $a_7 = -\frac{a_5}{7} = -\frac{a_1}{7 \cdot 5 \cdot 3}$ 8(9aq + 97) = 0n = 7 $a_q = -\frac{97}{9} = \frac{41}{9 \cdot 7.5.3}$ general, In $(-1)^{k}a_{1}a_{1}k_{1}$ $(-1)^{k}a_{1}a_{1}k_{1}$ $(-1)^{k}a_{1}a_{1}k_{1}$ 2.2.2.3.2.1 7.6.5.4.3.2.1 97(k=3)

When do we know differential equations have power series solutions? (Section 8.3)

Analyticity: A function F is said to be analytic at x=c if there arc real numbers ab, a, az, with $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ on some nonzero radius of convergence about X=C

$$e^{X} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} (radius = \infty)$$

$$n=0 \quad (-1)^{n} \frac{x^{n+1}}{x!} (radius = \infty)$$

$$sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n} x^{n+1}}{(2n+1)!}$$

$$\ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n} (r \cdot dius = 1)$$

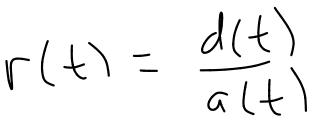
This shows ex, sin(x) are analytic at C=O and In(x) is analytic at (=1. In fact, ex and Sin(x) will be analytic for all values of C, but In(x) will only be analytic for ×70 ·

Back to standard form

(riven

a(t)y''(t)+b(t)y'(t)+((t)y(t)=d(t))

divide by a(t) and set $P(t) = \frac{b(t)}{a(t)}, q(t) = \frac{c(t)}{a(t)}$



Standard form! Y'' + p(t)y(t) + q(t)y(t)= L(f).

Ordinary and Singular Points

We say t=c is an Ordinary point of y'' + p(t)y' + q(t) if PJq are analytic at t=c (except possibly at t=c). We say t=c is a singular point if one of p or q is not analytic at t=c.

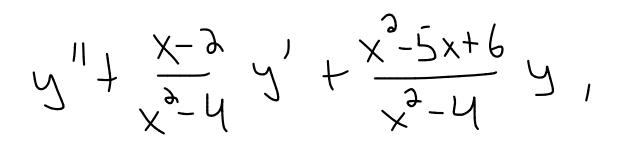
Example 3: Find all Singular points for $(x^{-4})y' + (x^{-3})y' + (x^{-5x+6})y = 0$ Divide by X²-U: $y'' + \frac{x-3}{x^{2}-4}y' + \frac{x-5x+6}{x^{2}-4}y = D$ $P(x) = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}(x+2)$

 $P(x) = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}(x+2)$ Only Singular point is X=-2. $q(x) = \frac{\chi^2 - 5\chi + L}{\chi^2 - 4} = \frac{(\chi - 2)(\chi - 3)}{(\chi - 3)(\chi + 2)}$ $=\frac{\chi-3}{\chi+3}$ except at x=2. Only singular point: X=-2

Regular and Irregular Singular Points

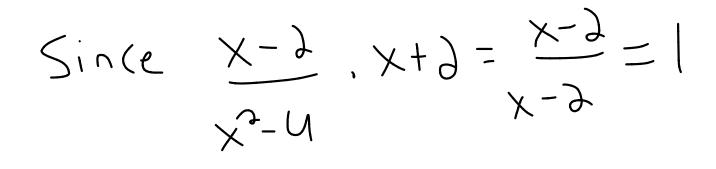
A singular point x=C for y"+p(x)y'+q(x) is called regular if (x-c)p(x) and (x-c) q(x) are analytic at x=c. Otherwise, X=C is called irregular

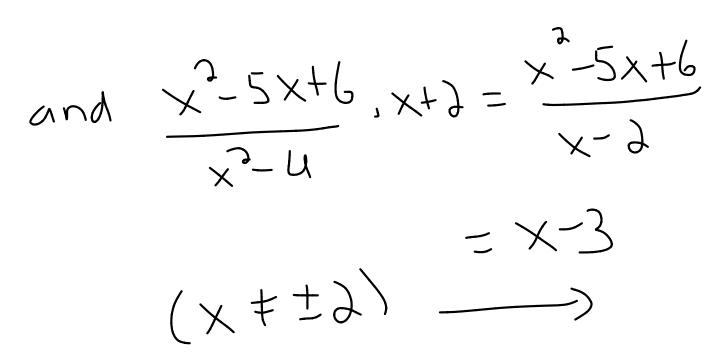
Example 4: For



we found that X = - 2

is a singular point.





We get that X = -d is a regular singular point since f(x) = 1 and g(x) = x - 3 are both analytic at x = -d.